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Quantum information and entropy squeezing of a two-level atom with a non-linear medium

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Abstract

In the language of quantum information theory we study the entropy squeezing of a two-level atom in a Kerr-like medium. A definition of squeezing is presented for this system, based on information theory. The utility of the definition is illustrated by examining the entropy squeezing of a two-level atom with a Kerr-like medium. The influence of the non-linear interaction of the Kerr medium, the atomic coherence and the detuning parameter on the properties of the entropy and squeezing of the atomic variables is examined.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Recently the new field of quantum information and computation has emerged, not only offering the potential of immense practical computing power, but also suggesting deep links between the well-established disciplines of quantum theory, information theory and computer science. Theoretically quantum computers can perform some types of calculations much faster than classical computers [1], but the technological difficulties of manipulating quantum information have so far prevented researchers from constructing a quantum computer which is able to perform useful tasks. The difficulty of building a quantum computer was greatly diminished when it was realized that a network of quantum phase gates operating in the product space of two qubits, single-bit rotations and single-bit phase shift gates can constitute a universal quantum computer [2]. The quantum phase gate simply gives the product state of two qubits a phase shift depending on the values of each qubit. In other words, the quantum phase gates perform the operation $|00\rangle, e^{i\alpha}|01\rangle, e^{i\beta}|10\rangle, e^{i\gamma}|11\rangle$ in the computational basis of the two qubits. Provided that $\alpha + \beta \neq \gamma \pmod{2\pi}$, a network of quantum phase gates supplemented with single-bit gates can mimic the operation of any other unitary operator acting on the qubits. Recently an implementation of a quantum phase gate has been demonstrated [3] utilizing Rydberg states and a photon in a microwave cavity. The field of quantum information and computing is based on manipulation of quantum coherent states [4]. The existing devices of quantum optics have been proposed as experimental implementation and employed to realize quantum computers. A scheme depending on applications of the displacing operator and propagating a laser beam in a non-linear Kerr medium has been proposed to perform quantum gates [5].

In the meantime squeezing states of light offer possibilities of improving the performance of optical devices since they can reduce fluctuations in one of the quadratures below the level associated with the vacuum states [6]. This situation is relevant for the optical communication networks as well as for many optical devices. Such a light has recently been used in a power-recycled interferometer [7] and in a phase-modulated signal-recycled interferometer [8], aiming to improve significantly the sensitivity of these devices. It has been shown that this light can be used to tune the resonant frequency of the cavity without actually moving the signal recycling mirror, or changing the bandwidth of the interferometer without substantially decreasing the sensitivity at the resonant frequency [8]. Also we may point out that squeezed light has been applied in quantum information theory, for example, in quantum teleportation [9, 10], cryptography [11, 12] and dense coding [13].

In this respect the security in quantum cryptography [14] relies on the uncertainty relation for field quadrature components of these states. Here we may refer to the experiments on quantum teleportation which have been successfully performed by means of two-mode squeezed vacuum states [15]. Also it is worthwhile mentioning that all experimental proposals for teleportation have involved dichotomic variables in $SU(2)$ with optical schemes accomplishing the Bell-operator measurement with low efficiency.

On the other hand, an optical medium exhibiting the frequency-independent Kerr effect is governed by a non-linear polarization vector P or a field-dependent susceptibility with components

$$P_i = \chi^{(1)}_{i,j} E_j + \chi^{(2)}_{i,j,k} E_j E_k + \chi^{(3)}_{i,j,k,l} E_j E_k E_l \qquad (1)$$

where E_i, E_j, E_k and E_l are the electric fields, $\chi^{(1)}_{i,j}$ gives rise to the familiar dispersion relation $\chi^{(2)}_{i,j,k}$ leads to a cubic nonlinearity which is responsible for three-wave mixing processes in parametric devices as well as second harmonic and subharmonic generations, while $\chi^{(3)}_{i,j,k,l}$ is responsible for four-wave mixing processes which have recently found considerable interest in connection with the phenomena of optical phase conjugation, real time holography, image correlation and different multiphoton spectroscopy techniques. A more familiar four-wave process is the stimulated Raman- and Brillouin-scattering, and the parametric coupling of Stokes and anti-Stokes radiation.

The behaviour of the electric fields representing the two qubits as they travel through the Kerr medium is given by the Hamiltonian

$$H = \frac{1}{2}\int \frac{1}{\mu_0} B^2 \,\mathrm{d}v + \frac{1}{2}\int \epsilon_0 E^2 \,\mathrm{d}v + \frac{1}{2}\int \chi E^2 \,\mathrm{d}v + \frac{3}{4}\int E P^{(3)}_{NL} \,\mathrm{d}v \qquad (2)$$

where μ_0 and ϵ_0 are the permeability of the non-magnetic material and the susceptibility of the electric field, respectively, B is the total magnetic field and $P^{(3)}_{NL}$ is the non-linear part of the polarization vector including frequency-dependent terms. We may simplify this complicated Hamiltonian by choosing the two frequencies of the electric fields to be nearly resonant with excitations of the medium [16], however exactly resonant photons will suffer from loss. Therefore it will be more convenient to keep the fields slightly detuned. Here we may refer to the experimental work by Sinatra *et al* [17], who used different energy transitions to couple two laser beams through a gas of ^{87}Rb where the photons in this case interact with a three-level system.

In fact the quantum phase gate can operate in the product space of the polarizations for two photons by using the optical Kerr effect, where the photons are made to interact as they pass through a material with a third-order non-linear susceptibility. In the presence of a superposition of electromagnetic waves at different frequencies and/or in different directions, these materials are used in four-wave mixing applications, such as frequency conversion, phase conjugation, etc, see above. It is also noted that in the presence of a wave at a single frequency, the refractive index of such materials is intensity dependent and gives rise to the phenomenon of self-focusing [18]. Therefore, when a superposition of waves is presented, the optical Kerr effect produces an interaction in which the intensity of one frequency component influences the index of refraction of another frequency component. As described by Mandel and Wolf in [19], this effect can be used to perform quantum non-demolition and back-action evading measurements, during which the intensity of one frequency component can be used to control the phase of another without altering the photon number of either component. Thus without loss of photon number, the frequency components can become entangled in a way that lends itself well to quantum computations. From the quantum information point of view, we shall consider the problem of entropy squeezing (von Neumann entropy) for a two-level atom interacting with a single mode in the presence of a Kerr-like medium. The organization of the paper is as follows: in section 2 we introduce our Hamiltonian model and give an exact expression for the density matrix $\hat{\rho}(t)$. In section 3 we employ the density matrix to investigate the properties of the entropy squeezing. Finally we devote section 4 to our discussion of the results.

2. The Hamiltonian model

Here we shall consider the Hamiltonian of a model that consists of a two-level atom interacting with a quantized radiation field taking into consideration an ideal cavity ($Q = \infty$) filled with a non-linear Kerr-like medium in the rotating wave approximation frame. We shall assume that the cavity mode is interacting with both the atom and the Kerr-like medium. Although in reality a cavity cannot be absolutely perfect, however the authors of [20] in their analysis have shown that, for a cavity with finite bandwidth at nonzero temperature T, the effects of the bandwidth and the temperature are negligible until time $t \sim 10^{-3}$ ($\lambda t = 30$) from the start of the interaction, provided that the values for Q and T are $Q = 2 \times 10^{10}$ and $T = 0.5$ K. Kerr effects can be observed by surrounding the atom by a non-linear medium inside a high-Q cavity [21].

The effective Hamiltonian model representing the interaction between a two-level atom and a single-mode cavity ($\hbar = c = 1$) in the presence of a Kerr-like medium in the rotating wave approximation can be written as

$$H_{\text{eff}} = \omega_c \hat{a}^\dagger \hat{a} + \frac{\omega_a}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + \chi \hat{a}^{\dagger 2}\hat{a}^2 + \lambda(\hat{a}^\dagger |g\rangle\langle e| + \hat{a}|e\rangle\langle g|) \tag{3}$$

where ω_c is the field frequency, ω_a is the transition frequency between the excited and the ground states of the atom and λ is the effective coupling constant. We denote by χ the dispersive part of the third-order nonlinearity of the Kerr-like medium, with the detuning parameter $\Delta = \omega_a - \omega_c$. We consider that at $t = 0$ the atom is initially in the coherent state $|\theta, \phi\rangle$ given by

$$|\theta, \phi\rangle = \cos(\theta/2)\,|e\rangle + \sin(\theta/2)\exp(-\mathrm{i}\phi)\,|g\rangle \tag{4}$$

where ϕ is the relative phase of the two atomic levels. For the excited state we take $\theta \to 0$ while for $\theta \to \pi$ the wavefunction describes the atom in the ground state. Further we assume that the field is initially in the coherent state, $|\alpha\rangle = \sum_n^\infty q_n\ |n\rangle$, where q_n describes the amplitude for the field mode in the state $|n\rangle$. Now if we take $\alpha = |\alpha|\mathrm{e}^{\mathrm{i}\beta}$ and consider that at $t = 0$ the field–atom system is decorrelated, then the initial density operator of the system is $\rho(0) = \rho_f(0) \otimes \rho_a(0)$, where $\rho_f(0) = |\alpha\rangle\langle\alpha|$ and $\rho_a(0) = |\theta, \phi\rangle\langle\theta, \phi|$ describe the initial densities for the field and the atom, respectively.

At any time $t > 0$ the density matrix of the system can be written as

$$\rho(t) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(a_n(t)a_m^*(t)\ |n, g\rangle\langle m, g| + b_n(t)b_m^*(t)\ |n, e\rangle\langle m, e| + a_n(t)b_m^*(t)\ |n, g\rangle\langle m, e| + b_n(t)a_m^*(t)\ |n, e\rangle\langle m, g|) \tag{5}$$

where the coefficients $a_n(t)$ and $b_n(t)$ are, respectively, given by

$$a_n(t) = \mathrm{e}^{-\mathrm{i}\lambda t\Gamma_n}\left[\sin(\theta/2)\mathrm{e}^{-\mathrm{i}\phi}q_n\left(\cos\lambda t\mu_n + \mathrm{i}W_n\frac{\sin\lambda t\mu_n}{\mu_n}\right) - \mathrm{i}\cos(\theta/2)q_{n-1}\frac{\sqrt{n}\sin\lambda t\mu_n}{\mu_n}\right] \tag{6}$$

$$b_n(t) = \mathrm{e}^{-\mathrm{i}\lambda t\Gamma_{n+1}}\left[\cos(\theta/2)q_n\cos\lambda t\mu_{n+1} - \mathrm{i}[W_{n+1}\ \cos(\theta/2)q_n + \sin(\theta/2)\mathrm{e}^{-\mathrm{i}\phi}q_{n+1}\sqrt{n+1}]\frac{\sin\lambda t\mu_{n+1}}{\mu_{n+1}}\right]. \tag{7}$$

The quantities μ_n, Γ_n and W_n in the above equations are

$$\mu_n = \sqrt{n + W_n^2} \qquad \Gamma_n = \frac{\chi}{\lambda}(n-1)^2 \qquad W_n = \frac{\Delta}{2\lambda} - \frac{\chi}{\lambda}(n-1). \tag{8}$$

Having obtained the density matrix $\hat{\rho}$, we are in a position to discuss the properties of the atom and the field. This will be seen in section 3.

3. Entropy squeezing

The quantum dynamics described by the Hamiltonian (3) leads to an entanglement between the field and the atom in the system under consideration. In this section we shall employ the uncertainty relation to study the squeezing entropy. Although the Heisenberg uncertainty relation cannot give us sufficient information on the atomic squeezing for some cases, however it can be used as a general criterion for the squeezing in terms of entropy of a two-level atom in the Jaynes–Cummings model.

The uncertainty relation for a two-level atom characterized by the Pauli operators S_x, S_y and S_z is given by

$$\Delta S_x\Delta S_y \geqslant \tfrac{1}{2}|\langle S_z\rangle| \tag{9}$$

where $\Delta S_\alpha = \sqrt{\langle S_\alpha^2\rangle - \langle S_\alpha\rangle^2}$.

Fluctuations in the component S_α of the atomic dipole are said to be squeezed if S_α satisfies the condition

$$V(S_\alpha) = \left(\Delta S_\alpha - \sqrt{\frac{|\langle S_z\rangle|}{2}}\right) < 0 \qquad \alpha = x \text{ or } y. \tag{10}$$

Recently in an even N-dimensional Hilbert space, the optimal entropic uncertainty relation for sets of $N + 1$ complementary observables with non-degenerate eigenvalues has been investigated [22]. This can be described by the inequality

$$\sum_{k=1}^{N+1} H(S_k) \geqslant \frac{N}{2}\ln\left(\frac{N}{2}\right) + \left(1 + \frac{N}{2}\right)\ln\left(1 + \frac{N}{2}\right) \tag{11}$$

where $H(S_k)$ represents the entropy of the variable S_k. On the other hand, for an arbitrary quantum state the probability distribution for N possible outcomes of measurements of the operator S_α is

$$P_i(S_\alpha) = \langle \Psi_{\alpha i} | \rho | \Psi_{\alpha i} \rangle \qquad \alpha = x, y, z \qquad i = 1, 2, \ldots, N \tag{12}$$

where $|\Psi_{\alpha i}\rangle$ is an eigenvector of the operator S_α such that

$$S_\alpha |\Psi_{\alpha i}\rangle = \lambda_{\alpha i} |\Psi_{\alpha i}\rangle \qquad \alpha = x, y, z \qquad i = 1, 2, \ldots, N. \tag{13}$$

The corresponding entropies are defined as

$$H(S_\alpha) = -\sum_{i=1}^{N} P_i(S_\alpha) \ln P_i(S_\alpha) \qquad \alpha = x, y, z. \tag{14}$$

Thus, to obtain the entropies of the atomic operators S_x, S_y and S_z for a two-level atom, with $N = 2$, one can use the reduced atomic density operator $\rho(t)$. For the present case we find that

$$\begin{aligned} H(S_x) &= -\left[\tfrac{1}{2} + \mathrm{Re}\{\rho_{ge}(t)\}\right] \ln\left[\tfrac{1}{2} + \mathrm{Re}\{\rho_{ge}(t)\}\right] - \left[\tfrac{1}{2} - \mathrm{Re}\{\rho_{ge}(t)\}\right] \ln\left[\tfrac{1}{2} - \mathrm{Re}\{\rho_{ge}(t)\}\right] \\ H(S_y) &= -\left[\tfrac{1}{2} + \mathrm{Im}\{\rho_{ge}(t)\}\right] \ln\left[\tfrac{1}{2} + \mathrm{Im}\{\rho_{ge}(t)\}\right] - \left[\tfrac{1}{2} - \mathrm{Im}\{\rho_{ge}(t)\}\right] \ln\left[\tfrac{1}{2} - \mathrm{Im}\{\rho_{ge}(t)\}\right] \end{aligned} \tag{15}$$

and

$$H(S_z) = -\rho_{gg}(t) \ln \rho_{gg}(t) - \rho_{ee}(t) \ln \rho_{ee}(t) \tag{16}$$

where the quantities $\rho_{gg}(t)$, $\rho_{ge}(t)$, $\rho_{ee}(t)$ and $\rho_{eg}(t) = \rho^*_{ge}(t)$ are determined from the relations

$$\begin{aligned} \rho_{gg}(t) = \sum_{n=0}^{\infty} P_n \Bigg[& \sin^2(\theta/2) \left(\cos^2 \lambda t \mu_n + \frac{W_n^2 \sin^2 \lambda t \mu_n}{\mu_n^2} \right) + \left(\frac{n \sin \lambda t \mu_n \cos(\theta/2)}{|\alpha| \mu_n} \right)^2 \\ & + \frac{2n \sin \lambda t \mu_n \sin\theta}{|\alpha| \mu_n} \left(\cos \lambda t \mu_n \sin(\phi - \beta) - \frac{W_n \sin \lambda t \mu_n \cos(\phi - \beta)}{\mu_n} \right) \Bigg] \end{aligned} \tag{17}$$

and

$$\begin{aligned} \rho_{ee}(t) = \sum_{n=0}^{\infty} P_n \Bigg[& \cos^2(\theta/2) \left(\cos^2 \lambda t \mu_{n+1} + \frac{W_{n+1}^2 \sin^2 \lambda t \mu_{n+1}}{\mu_{n+1}^2} \right) \\ & + \left(\frac{|\alpha| \sin \lambda t \mu_{n+1} \sin(\theta/2)}{\mu_{n+1}} \right)^2 - \frac{2|\alpha| \sin \lambda t \mu_{n+1} \sin(\theta)}{\mu_{n+1}} \\ & \times \left(\cos \lambda t \mu_{n+1} \sin(\phi - \beta) - \frac{W_{n+1} \sin \lambda t \mu_{n+1} \cos(\phi - \beta)}{\mu_{n+1}} \right) \Bigg] \end{aligned} \tag{18}$$

while

$$\rho_{ge}(t) = R(t) - \mathrm{i}V(t) \tag{19}$$

with

$$\begin{aligned} R(t) = \sum_{n=0}^{\infty} P_n \Bigg[& \left(R_2(t) + \frac{R_1(t) \sin\theta}{2} \right) \cos \lambda t (\Gamma_{n+1} - \Gamma_n) \\ & + \left(V_2(t) + \frac{V_1(t) \sin\theta}{2} \right) \sin \lambda t (\Gamma_{n+1} - \Gamma_n) \Bigg] \end{aligned} \tag{20}$$

and

$$V(t)=\sum_{n=0}^{\infty}P_n\left[\left(V_2(t)+\frac{V_1(t)\sin\theta}{2}\right)\cos\lambda t(\Gamma_{n+1}-\Gamma_n)\right.$$
$$\left.-\left(R_2(t)+\frac{R_1(t)\sin\theta}{2}\right)\sin\lambda t(\Gamma_{n+1}-\Gamma_n)\right]. \tag{21}$$

The expressions for $R_i(t)$ and $V_i(t)$, $i = 1, 2$, in the above equations are given by

$$R_1(t)=\left(\cos\lambda t\mu_n\cos\lambda t\mu_{n+1}-\frac{W_nW_{n+1}\sin\lambda t\mu_n\sin\lambda t\mu_{n+1}}{\mu_n\mu_{n+1}}\right)\cos\phi$$
$$+\left(\frac{W_n\sin\lambda t\mu_n\cos\lambda t\mu_{n+1}}{\mu_n}+\frac{W_{n+1}\sin\lambda t\mu_{n+1}\cos\lambda t\mu_n}{\mu_{n+1}}\right)\sin\phi$$
$$+\frac{n\sin\lambda t\mu_n\sin\lambda t\mu_{n+1}\cos(\phi-2\beta)}{\mu_n\mu_{n+1}} \tag{22}$$

$$R_2(t)=\frac{|\alpha|\sin\lambda t\mu_{n+1}\sin^2(\theta/2)}{\mu_{n+1}}\left(\cos\lambda t\mu_n\sin\beta-\frac{W_n\sin\lambda t\mu_n\cos\beta}{\mu_n}\right)$$
$$-\frac{n\sin\lambda t\mu_n\cos^2(\theta/2)}{|\alpha|\mu_n}\left(\cos\lambda t\mu_{n+1}\sin\beta-\frac{W_{n+1}\sin\lambda t\mu_{n+1}\cos\beta}{\mu_{n+1}}\right) \tag{23}$$

$$V_1(t)=\left(\cos\lambda t\mu_n\cos\lambda t\mu_{n+1}-\frac{W_nW_{n+1}\sin\lambda t\mu_n\sin\lambda t\mu_{n+1}}{\mu_n\mu_{n+1}}\right)\sin\phi$$
$$-\left(\frac{W_n\sin\lambda t\mu_n\cos\lambda t\mu_{n+1}}{\mu_n}+\frac{W_{n+1}\sin\lambda t\mu_{n+1}\cos\lambda t\mu_n}{\mu_{n+1}}\right)\cos\phi$$
$$-\frac{n\sin\lambda t\mu_n\sin\lambda t\mu_{n+1}\sin(\phi-2\beta)}{\mu_n\mu_{n+1}} \tag{24}$$

$$V_2(t)=\frac{n\sin\lambda t\mu_n\cos^2(\theta/2)}{|\alpha|\mu_n}\left(\cos\lambda t\mu_{n+1}\cos\beta+\frac{W_{n+1}\sin\lambda t\mu_{n+1}\sin\beta}{\mu_{n+1}}\right)$$
$$-\frac{|\alpha|\sin\lambda t\mu_{n+1}\sin^2(\theta/2)}{\mu_{n+1}}\left(\cos\lambda t\mu_n\cos\beta+\frac{W_n\sin\lambda t\mu_n\sin\beta}{\mu_n}\right). \tag{25}$$

For a two-level atom, where $N = 2$, we have

$$0\leqslant H(S_\alpha)\leqslant\ln 2 \tag{26}$$

and hence, the entropies of the operators S_x, S_y and S_z will satisfy the inequality

$$H(S_x)+H(S_y)\geqslant 2\ln 2-H(S_z). \tag{27}$$

In other words if we define

$$\delta H(S_\alpha)=\exp[H(S_\alpha)] \tag{28}$$

the inequality (27) can be written as

$$\delta H(S_x)\delta H(S_y)\geqslant\frac{4}{\delta H(S_z)}. \tag{29}$$

Now if $\delta H(S_\alpha) = 1$, the atom will be in a pure state; however, when $\delta H(S_\alpha)$ takes the value 2, the atom will be in a completely mixed state. Since the quantities $\delta H(S_x)$ and $\delta H(S_y)$ only measure the uncertainties of the atomic polarization components S_x and S_y, respectively, it is clear from the entropic uncertainty relation (11) that it is impossible to simultaneously have complete information about the observables S_x and S_y. Now let us define here the squeezing of

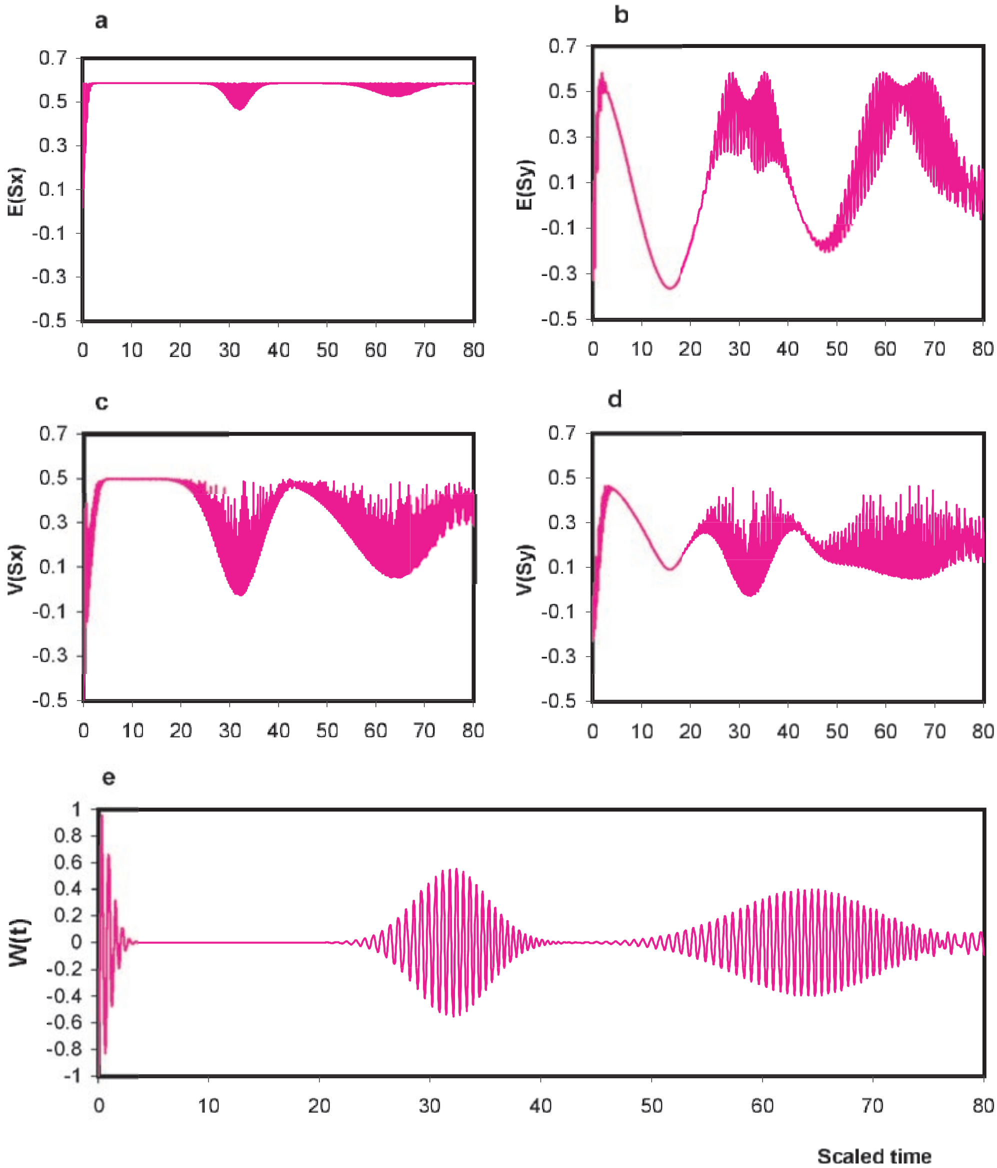

Figure 1. The time evolution of the squeezing factors of a two-level atom interacting with a single mode in the absence of a Kerr-like medium. The atom is initially in the excited state and the field is in the coherent state with the initial average photon number $\bar{n} = 25$. (*a*) The entropy squeezing factor $E(S_x)$; (*b*) the entropy squeezing factor $E(S_y)$; (*c*) the variance squeezing factor $V(S_x)$; (*d*) the variance squeezing factor $V(S_y)$ and (*e*) the time evolution of the atomic inversion under the same conditions.

the atom using the inequality (27), called entropy squeezing, which has in fact received a little attention in the literature. The fluctuations in component S_α ($\alpha = x$ or y) of the atomic dipole are said to be 'squeezed in entropy' if the information entropy $H(S_\alpha)$ of S_α satisfies the condition

$$E(S_\alpha) = \left(\delta H(S_\alpha) - \frac{2}{\sqrt{|\delta H(S_z)|}}\right) < 0 \qquad \alpha = x \text{ or } y. \tag{30}$$

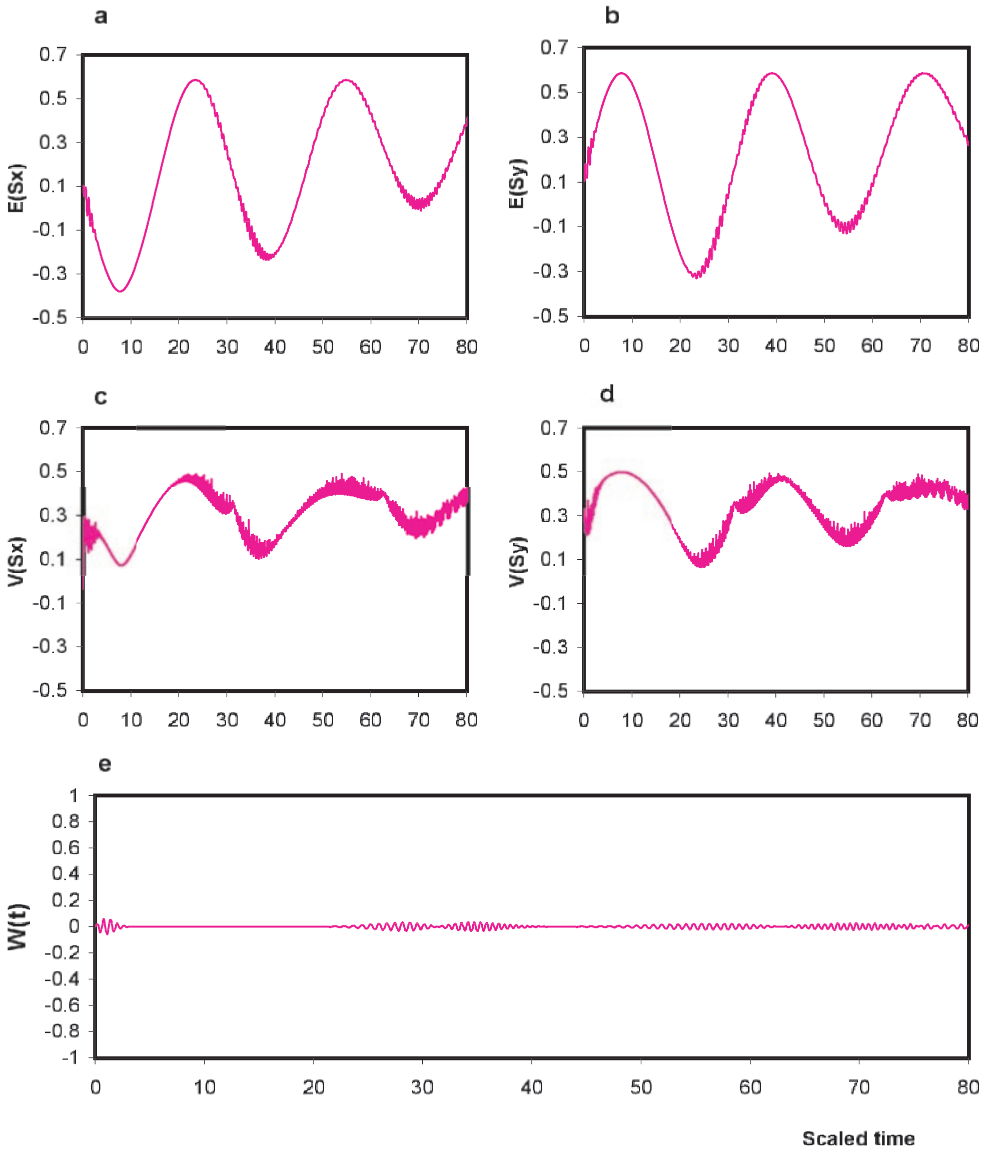

Figure 2. The time evolution of the squeezing factors of a two-level atom interacting with a single mode in the absence of a Kerr-like medium. The atom is initially in the superposition state, $\theta = \pi/2$, $\phi = \pi/4$, and the field is in a coherent state with $\bar{n} = 25$ and $\beta = \pi/4$. (*a*) The entropy squeezing factor $E(S_x)$; (*b*) the entropy squeezing factor $E(S_y)$; (*c*) the variance squeezing factor $V(S_x)$; (*d*) the variance squeezing factor $V(S_y)$ and (*e*) the time evolution of the atomic inversion under the same conditions.

Employing the results obtained here we shall be able to discuss the entropy squeezing. This will be done in section 4.

4. Discussion and conclusion

On the basis of the analytical solution presented in section 3, we shall examine the temporal evolutions of the entropy squeezing (Von Neumann entropy) and variance squeezing.

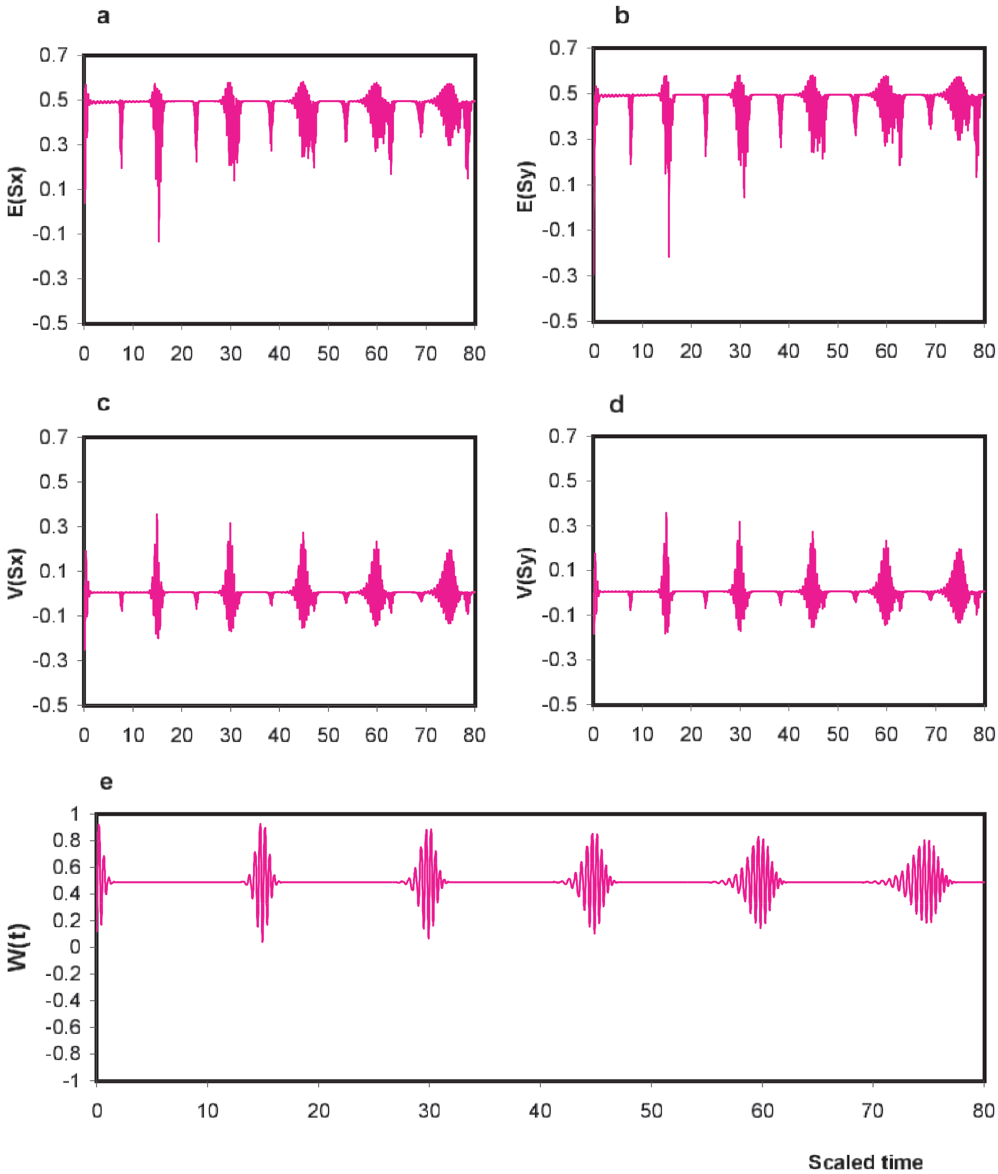

Figure 3. The time evolution of the squeezing factors of a two-level atom interacting with a single mode with a Kerr-like medium, $\chi/\lambda = 0.5$, and in the presence of the detuning parameter $\Delta/\lambda = 10$. The atom is initially in the superposition state, $\theta = \pi/2$, $\phi = \pi/4$, and the field is in a coherent state with $\bar{n} = 25$ and $\beta = \pi/4$. (*a*) The entropy squeezing factor $E(S_x)$; (*b*) the entropy squeezing factor $E(S_y)$; (*c*) the variance squeezing factor $V(S_x)$; (*d*) the variance squeezing factor $V(S_y)$ and (*e*) the time evolution of the atomic inversion under the same conditions.

The time evolutions of the squeezing factors $E(S_x)$, $E(S_y)$, $V(S_x)$ and $V(S_y)$ are shown in figures 1(*a*)–(*d*) but the atomic inversion $\langle\sigma_z(t)\rangle$ is plotted in figure 1(*e*), for an atom initially in the excited state ($\theta = 0$) with the mean photon number $\bar{n} = 25$, the relative phase $\beta = 0$ and in the absence of the Kerr medium and detuning parameter. Figures 1(*a*) and (*c*) predict no squeezing in the variable S_x when the atom is initially in the excited state, but figures 1(*b*) and (*d*) present a great difference between $E(S_y)$ and $V(S_y)$: $E(S_y)$ shows entropy squeezing

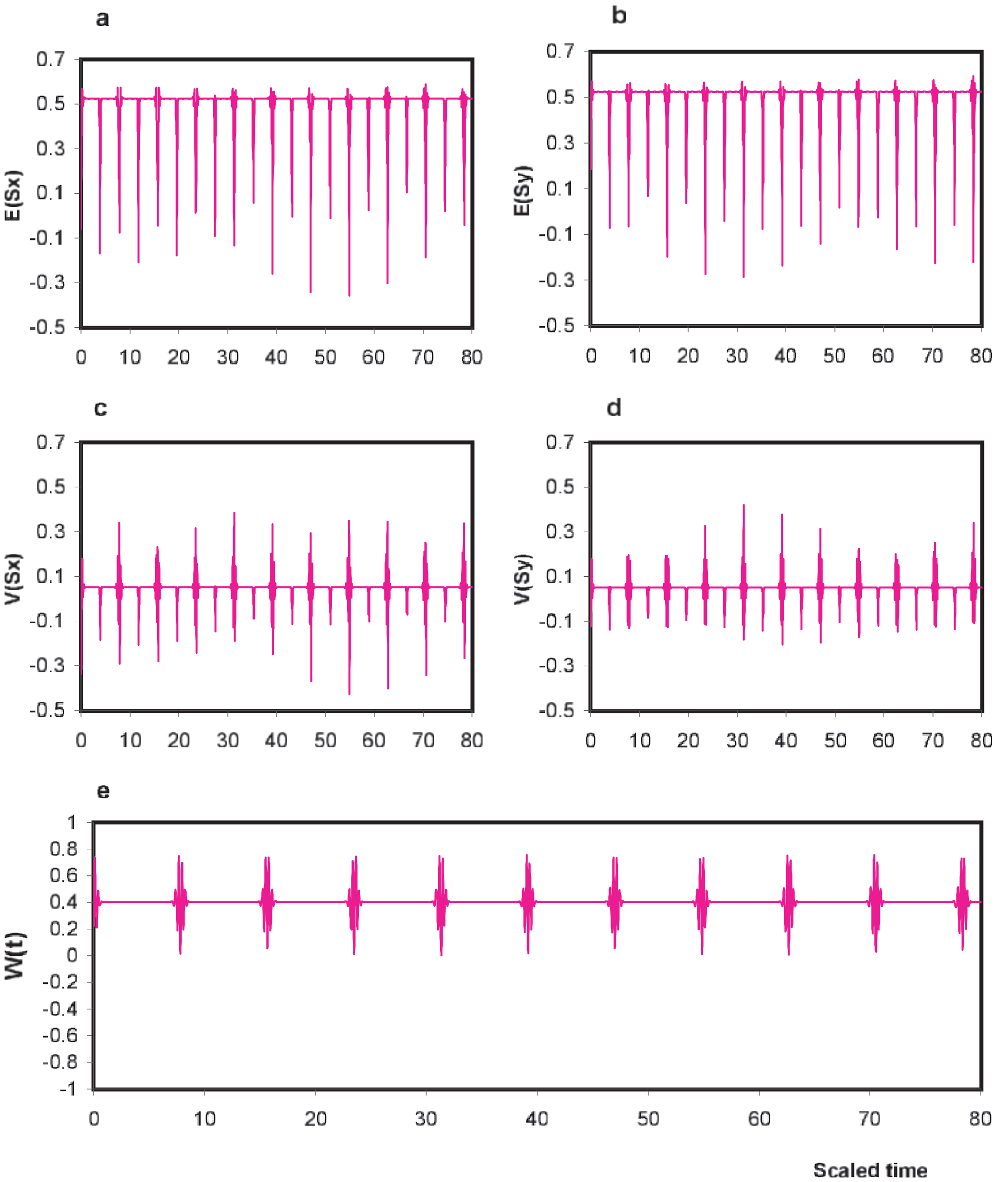

Figure 4. The time evolution of the squeezing factors of a two-level atom interacting with a single mode with a Kerr-like medium, $\chi/\lambda = 0.2$, and in the absence of the detuning parameter $\Delta/\lambda = 0$. The atom is initially in the superposition state, $\theta = \pi/2, \phi = \pi/4$, and the field is in a coherent state with $\bar{n} = 25$ and $\beta = \pi/4$. (*a*) The entropy squeezing factor $E(S_x)$; (*b*) the entropy squeezing factor $E(S_y)$; (*c*) the variance squeezing factor $\Gamma(S_x)$; (*d*) the variance squeezing factor $V(S_y)$ and (*e*) the time evolution of the atomic inversion under the same conditions.

during the collapse for the atomic inversion $W(t)$ as figure 1(*b*) exhibits, while $V(S_y)$ predicts variance squeezing in a short duration during the atomic inversion $W(t)$ revival, see figure 1(*e*).

Figure 1(*b*) shows that at half the revival time where $t = t_R/2 = \pi\sqrt{\frac{\bar{n}}{\lambda}} = \frac{5\pi}{\lambda}$, optimal entropy squeezing is attained, because at $t = t_R/2$ the atom has achieved an almost pure state $|\Psi_A(t_R/2)\rangle \simeq \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ at $\theta = \pi/2$ and $\phi = \pi/2$. This state is just an eigenstate

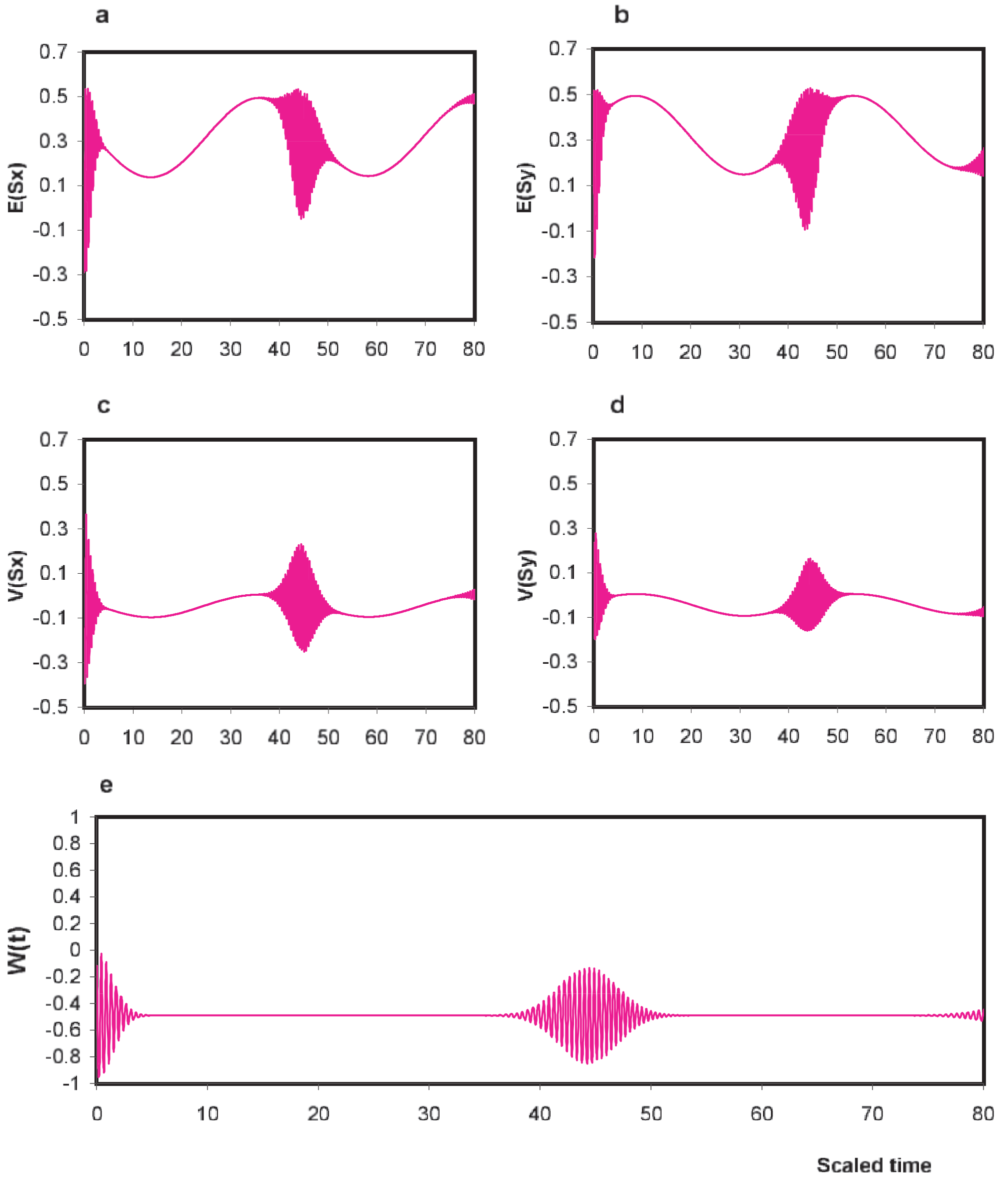

Figure 5. The time evolution of the squeezing factors of a two-level atom interacting with a single mode in the absence of a Kerr-like medium $\chi/\lambda = 0$ and in the presence of the detuning parameter $\Delta/\lambda = 10$. The atom is initially in the superposition state, $\theta = \pi/2$, $\phi = \pi/4$, and the field is in a coherent state with $\bar{n} = 25$ and $\beta = \pi/4$. (*a*) The entropy squeezing factor $E(S_x)$; (*b*) the entropy squeezing factor $E(S_y)$; (*c*) the variance squeezing factor $\Gamma(S_x)$; (*d*) the variance squeezing factor $V(S_y)$ and (*e*) the time evolution of the atomic inversion under the same conditions.

of the atomic operator S_y, then we have $\Delta S_y = 0$ is the smallest possible value as seen in figure 1(*d*) and does not exhibit any variance squeezing around this time, since the atomic inversion satisfies $\langle\sigma_z(t)\rangle = 0$ at $t = t_R/2$.

To realize the effect of the atomic superposition on the entropy squeezing and variance squeezing, we set $\theta = \pi/2$, $\phi = \pi/4$ and the field in a coherent state with $\bar{n} = 25$ and $\beta = \pi/4$. It is obvious from figures 2(*a*) and (*b*) that the entropies for the quadratures S_x and S_y

show alternating squeezing whereas $V(S_x)$ and $V(S_y)$, as illustrated in figures 3(*c*) and (*d*), respectively, display no variance squeezing. In figure 2(*b*) we see that entropy squeezing occurs where the atomic inversion is close to zero. This is almost the case of a trapped coherent state [23].

The effect of a Kerr medium on the squeezing of the entropy and variances is depicted in figures 3 and 4. We take the same parameters as in figure 2 and put $\chi/\lambda = 0.2$. The effect of a Kerr medium on the atomic occupation number results in inhibiting energy in the atomic system. The more χ/λ increases, the higher the mean values for $W(t)$ as shown in figure 3(*e*). Also it results in faster oscillations of the atomic inversion. This is also reflected in the behaviour of both entropy and variance squeezing quantities, $E(S_x)$ and $E(S_y)$; periodic variance squeezing is also observed in all of these quantities.

The detuning effect results in elongating the revival time $T_R = 2\pi\sqrt{\bar{n} + \Delta^2/4}$ as can be seen in figure 5. Also the atomic system loses some of its energy to the system as can be shown from the mean value for $W(t)$ that attains a lower value than that in the case of resonance. Squeezing of all quantities appears for this case; however, the amount of squeezing is not as pronounced as in the case of resonance.

Thus we have shown in the above sections that in our system the effect of a Kerr-like medium on the entropy is negative and the effect of detuning on the atomic variable squeezing is positive. This emphasizes the fact that the atomic coherence has a remarkable effect on the squeezing of the entropy, and the system of the Jaynes–Cumming model with a Kerr-like medium can have a potential application in the field of quantum information.

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